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Evaluation of Some Finite Integral Formulas Involving a Product of $\bar{\mathcal{H}}$-Function

Sunil Kumar Sharma[1] and Ashok Singh Shekhawat[2]
Research Scholar, Department of Mathematics[1]
Department of Mathematics[2]
Suresh Gyan Vihar University, Jagatpura, Jaipur-302025, Rajasthan, India[1, 2]
Email: sunil94147@gmail.com[1], *csmaths2004@yahoo.com*[2]

Abstract- In this paper, we study some finite integral formulas pertaining to the $\bar{\mathcal{H}}$-function, proposed by Inayat- Hussain. During the course of finding, we establish various integral results involving $\bar{\mathcal{H}}$-function multiplied with the help of an integral due to Lavoie and Trottier, certain integral due to Oberhettinger, algebraic functions and specials functions. These integrals relations are unified in nature and act as key formulas from which we can obtain large number of simpler functions and polynomials.

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Index Terms:-$\bar{\mathcal{H}}$-function, Lavoie-Trottie integrals, Certain integral due to Oberhettinger, Bessel Maitland function $\mathcal{J}_v^\mu(z)$, General class of polynomials $\mathcal{S}_n^m[x]$, Jecobi polynomials $\mathfrak{P}_n^{(\alpha,\beta)}$,Legendre function, Hypergeometric functions.

1. INTRODUCTION

Especially, in recent decades, several generalized of the well known special functions have been studied by different authors. A large number of integral formulas of a variety of special functions have been developed by many authors (See, e.g. [3-4], [7], [8], [17-18]). Those integrals involving Bessel-function, Hypergeometric function, General class of polynomials etc. are of great importance since they are used in applied physics and many branches of engineering.

The $\bar{\mathcal{H}}$-function [1-2], is a new generalization of well known Fox's H-function [11]. The $\bar{\mathcal{H}}$-function will be defined and represented as follows [10]:

$$\bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}[z] = \bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}\left[z\left|\begin{matrix}(a_j,\alpha_j;\mathcal{A}_j)_{1,\mathcal{N}},(a_j,\alpha_j)_{\mathcal{N}+1,\mathcal{P}}\\(b_j,\beta_j)_{1,\mathcal{M}},(b_j,\beta_j;\mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_{\ell}\Xi(\zeta)z^{-\zeta}\,d\zeta, \quad (1)$$

where $i=\sqrt{-1}$ and

$$\Xi(\zeta) = \frac{\prod_{j=1}^{\mathcal{M}}\Gamma(b_j-\beta_j\zeta)\prod_{j=1}^{\mathcal{N}}\{\Gamma(1-a_j+\alpha_j\zeta)\}^{\mathcal{A}_j}}{\prod_{j=\mathcal{M}+1}^{\mathcal{Q}}\{\Gamma(1-b_j+\beta_j\zeta)\}^{\mathcal{B}_j}\prod_{j=\mathcal{N}+1}^{\mathcal{P}}\Gamma(a_j-\alpha_j\zeta)} \quad (2)$$

which contains fractional powers of some of the gamma function. Here, and throughout the paper a_j $(j=\overline{1,\mathcal{P}})$ and b_j $(j=\overline{1,\mathcal{Q}})$ are complex parameters, $\alpha_j \geq 0$ $(j=\overline{1,\mathcal{P}})$, $\mathcal{B}_j \geq 0$ $(j=\overline{1,\mathcal{Q}})$ (not all zero simultaneously) and the exponents $\mathcal{A}_j(j=\overline{1,\mathcal{N}})$ and $\mathcal{B}_j(j=\overline{\mathcal{M}+1,\mathcal{Q}})$ can take on non-integer values.

The contour in (1) is imaginary axis $\mathcal{R}(\zeta)=0.$ It is suitably indented order to avoid the singularities of the gamma functions and to keep those singularities on appropriate sides. Again, for $\mathcal{A}_j(j=\overline{1,\mathcal{N}})$ not an integer, the poles of the gamma functions of numerator in (2) are converted to branch points. However, a long as there is no coincidence o poles from any $\Gamma(b_j-\beta_j\zeta)$ $(j=\overline{1,\mathcal{M}})$ and $\Gamma(1-a_j+\alpha_j\zeta)$ $(j=\overline{1,\mathcal{N}})$ pair, the branch cuts can be chosen so that the path of integration be distorted in the useful manner. For the sake brevity

$$T = \sum_{j=1}^{\mathcal{M}} \beta_j + \sum_{j=1}^{\mathcal{N}} |\mathcal{A}_j \alpha_j| - \sum_{j=1}^{\mathcal{Q}} |\mathcal{B}_j \beta_j|$$

$$- \sum_{j=\mathcal{N}+1}^{\mathcal{P}} \alpha_j > 0 \tag{3}$$

and $|\arg z| < \frac{1}{2} T\pi$.

Also, form Inayat-Hussain [9], it follows that

$\bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}[z] = O(|z|^{\zeta^*})$ for small z, where

$$\zeta^* = \min_{1 \le j \le \mathcal{M}} \left[\mathcal{R}\left(\frac{b_j}{\beta_j}\right) \right] \tag{4}$$

and $\bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}[z] = O(|z|^{\zeta^*})$ for large z, where

$$\zeta^* = \max_{1 \le j \le \mathcal{N}} \left[\mathcal{R}\left(\frac{a_j - 1}{\alpha_j}\right) \right] \tag{5}$$

Remark:-When the exponents $\mathcal{A}_j = \mathcal{B}_j = 1 \ \forall$ i and j, the $\bar{\mathcal{H}}$-function reduces to the familier Fox's H-function [12].

The generalized hypergeometric function ${}_{\mathfrak{p}}\mathfrak{F}_{\mathfrak{q}}$ with $\mathfrak{p}$ numerator parameters $a_1,\ldots,a_{\mathfrak{p}}$ such that $a_j \in \mathbb{C}$ $(j = 1, \ldots, \mathfrak{p})$ and $\mathfrak{q}$ denominator parameters $b_1, \ldots, b_{\mathfrak{q}}$ such that $b_j \in \mathbb{C}$ $(j = 1, \ldots, \mathfrak{q};\ z_0^- = z \cup \{0\} = \{0, -1, -2, \ldots\})$ is defined by [19] as:

$${}_{\mathfrak{p}}\mathfrak{F}_{\mathfrak{q}}\left[\begin{matrix} a_1, \ldots, a_{\mathfrak{p}}; \\ b_1, \ldots, b_{\mathfrak{q}}; \end{matrix} z\right] = {}_{\mathfrak{p}}\mathfrak{F}_{\mathfrak{q}}[a_1, \ldots, a_{\mathfrak{p}};\ b_1, \ldots, b_{\mathfrak{q}}; z]$$
$$= \sum_{r=0}^{\infty} \frac{(a_1)_r \ldots (a_{\mathfrak{p}})_r}{(b_1)_r \ldots (b_{\mathfrak{q}})_r} \frac{z^r}{r!} \tag{6}$$

The series occurring in (6) is convergent for all z (real or complex) when $\mathfrak{p} \le \mathfrak{q}$. If $\mathfrak{p} = \mathfrak{q} + 1$, then the series is convergent for $|z| \le 1$.

Moreover, with $\mathfrak{p} = \mathfrak{q} + 1$, the series (6) is

(i) Absolutely convergent for $|z| = 1$, if $\mathcal{R}(\Psi) > 0$, where

$$\Psi = \sum_{j=0}^{\mathfrak{q}} b_j - \sum_{j=0}^{\mathfrak{p}} a_j$$

(ii) Conditionally convergent for $|z| = 1, |z| \ne 1$, if $-1 < \mathcal{R}(\Psi) \le 0$ and

(iii) Divergent for $|z| = 1$, if $\mathcal{R}(\Psi) \le -1$.

If $\mathfrak{p} > q + 1$, the series never convergent except when z=0, and the function is only defined when the series terminates.

It is evident that, for every such hypergeometric identity, we can easily evaluate a number of finite integrals involving hypergeometirc functions.

A detailed of account of the functions ${}_2\mathfrak{F}_1$, ${}_1\mathfrak{F}_1$ and ${}_{\mathfrak{p}}\mathfrak{F}_{\mathfrak{q}}$ can be found in the standard works, by Rainville [19].

Also, $(\lambda)_n$ is the Pochhammer symbol defined (for $\lambda \in C$) by

$$(\lambda)_n = \begin{cases} 1 & (n = 0) \\ \lambda\,(\lambda + 1) \ldots (\lambda + n - 1) & (n \in N) \end{cases}$$
$$= \frac{\Gamma(\lambda + n)}{\Gamma(\lambda)} \ \ (\lambda \in \mathbb{C}/z_0^-) \tag{7}$$

1. NEW CLASS OF INTEGRALS INVOLVING PRODUCT OF $\bar{\mathcal{H}}$-FUNCTION

In this section, two integral formulas involving $\bar{\mathcal{H}}$-function are established. For the present investigation, we need the following result for Lavoie and Trottier [5]:

$$\int_0^1 x^{p-1}\ (1-x)^{2q-1} \left(1 - \frac{x}{3}\right)^{2p-1} \left(1 - \frac{x}{4}\right)^{q-1} dx$$
$$= \left(\frac{2}{3}\right)^{2p} \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \tag{8}$$

where $\mathcal{R}(p) > 0, \mathcal{R}(q) > 0$.

1.1. *First Integral*

$$\mathfrak{T}_1 = \int_0^1 x^{\eta+\sigma-1}(1-x)^{2\eta-1} \left(1 - \frac{x}{3}\right)^{2(\eta+\sigma)-1}$$
$$\left(1 - \frac{x}{4}\right)^{\eta-1} \times \bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}\left[y\ \left(1 - \frac{x}{4}\right)\ (1-x)^2\right]\ dx$$
$$= \frac{1}{2\pi i} \int_{\ell} \Xi\,(\zeta)\, y^{\zeta} \left[\int_0^1 x^{\eta+\sigma-1}\ (1-x)^{2(\eta+\zeta)-1} \right.$$
$$\left. \left(1 - \frac{x}{3}\right)^{2(\eta+\sigma)-1} \left(1 - \frac{x}{4}\right)^{\zeta+\eta-1} dx \right] d\zeta$$

Now using (2) and (8), we get

$$\mathfrak{T}_1 = \left(\frac{2}{3}\right)^{2(\eta+\sigma)} \Gamma\,(\eta+\sigma) \times \frac{1}{2\pi i}$$

$$\int_{\ell} \frac{\prod_{j=1}^{\mathcal{M}} \Gamma(b_j - \beta_j\zeta) \prod_{j=1}^{\mathcal{N}} \{\Gamma(1-a_j+\alpha_j\zeta)\}^{\mathcal{A}_j}}{\prod_{j=\mathcal{M}+1}^{\mathcal{Q}} \{\Gamma(1-b_j+\beta_j\zeta)\}^{\mathcal{B}_j} \prod_{j=\mathcal{N}+1}^{\mathcal{P}} \Gamma(a_j - \alpha_j\zeta)}$$

$$\times \frac{\Gamma(\eta+\zeta)}{\Gamma(2\eta+\sigma+\zeta)} \; y^{\zeta}\, d\zeta$$

which upon using (1), we get

$$\mathfrak{T}_1 = \left(\frac{2}{3}\right)^{2(\eta+\sigma)} \Gamma(\eta+\sigma) \times \bar{\mathcal{H}}_{\mathcal{P}+1,\mathcal{Q}+1}^{\mathcal{M},\mathcal{N}+1}$$

$$\left[y \left| \begin{matrix} (1-\eta, 1; 1), (a_j, \alpha_j; \mathcal{A}_j)_{1,\mathcal{N}}, (a_j, \alpha_j)_{\mathcal{N}+1,\mathcal{P}} \\ (b_j, \beta_j)_{1,\mathcal{M}}, (1-2\eta-\sigma, 1; 1), (b_j, \beta_j; \mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}} \end{matrix} \right. \right] \tag{9}$$

1.2. *Second Integral*

$$\mathfrak{T}_2 = \int_0^1 x^{\eta-1} (1-x)^{2(\eta+\sigma)-1} \left(1-\frac{x}{3}\right)^{2\eta-1}$$

$$\left(1-\frac{x}{4}\right)^{\eta+\sigma-1} \times \bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}\left[xy \left(1-\frac{x}{3}\right)^2\right] dx$$

$$= \frac{1}{2\pi i}\int_{\ell} \Xi(\zeta)\, y^{\zeta} \left\{ \int_0^1 x^{\eta+\zeta-1} (1-x)^{2(\eta+\sigma)-1} \right.$$

$$\left. \left(1-\frac{x}{3}\right)^{2(\zeta+\eta)-1} \left(1-\frac{x}{4}\right)^{\eta+\sigma-1} dx \right\} d\zeta$$

Now using (2) and (8), we have

$$\mathfrak{T}_2 = \left(\frac{2}{3}\right)^{2\eta} \Gamma(\eta+\sigma) \times \frac{1}{2\pi i}$$

$$\int_{\ell} \frac{\prod_{j=1}^{\mathcal{M}} \Gamma(b_j - \beta_j\zeta) \prod_{j=1}^{\mathcal{N}} \{\Gamma(1-a_j+\alpha_j\zeta)\}^{\mathcal{A}_j}}{\prod_{j=\mathcal{M}+1}^{\mathcal{Q}} \{\Gamma(1-b_j+\beta_j\zeta)\}^{\mathcal{B}_j} \prod_{j=\mathcal{N}+1}^{\mathcal{P}} \Gamma(a_j - \alpha_j\zeta)}$$

$$\times \frac{\Gamma(\eta+\zeta)}{\Gamma(2\eta+\sigma+\zeta)} \left(\frac{4y}{9}\right)^{\zeta} d\zeta$$

$$\mathfrak{T}_2 = \left(\frac{2}{3}\right)^{2\eta} \Gamma(\eta+\sigma) \times \bar{\mathcal{H}}_{\mathcal{P}+1,\mathcal{Q}+1}^{\mathcal{M},\mathcal{N}+1}$$

$$\left[\frac{4y}{9} \left| \begin{matrix} (1-\eta, 1; 1), (a_j, \alpha_j; \mathcal{A}_j)_{1,\mathcal{N}}, (a_j, \alpha_j)_{\mathcal{N}+1,\mathcal{P}} \\ (b_j, \beta_j)_{1,\mathcal{M}}, (1-2\eta-\sigma, 1; 1), (b_j, \beta_j; \mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}} \end{matrix} \right. \right] \tag{10}$$

3. INTEGRAL FORMULA INVOLVING PRODUCT OF $\bar{\mathcal{H}}$-FUNCTION WITH BESSEL MAITLAND FUNCTIONS

The Bessel Maitland (also known as Wright generalized Bessel function) defined as following [13]:

$$\mathcal{J}_{\eta}^{\lambda}(x) = \Theta(\lambda, \eta+1; x)$$

$$= \sum_{l=0}^{\infty} \frac{1}{\Gamma(\lambda l+\eta+1)} \frac{(-x)^l}{l!} \tag{11}$$

1.3. *Third Integral*

$$\mathfrak{T}_3 = \int_0^{\infty} x^{\rho}\, \mathcal{J}_{\eta}^{\lambda}(x)\; \bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}\,(z\,x^{\sigma})\, dx$$

$$= \frac{1}{2\pi i}\int_{\ell} \Xi(\zeta)\, z^{\zeta} \left\{ \int_0^{\infty} x^{\rho+\sigma\zeta}\; \mathcal{J}_{\eta}^{\lambda}(x)\, dx \right\} d\zeta$$

Next, using the following formula [9]:

$$\int_0^{\infty} x^{\rho}\, \mathcal{J}_{\eta}^{\lambda}(x)\, dx = \frac{\Gamma(\rho+1)}{\Gamma(1+\eta-\lambda-\lambda\rho)}$$

$$(\Re(\rho) > -1, 0 < \lambda < 1) \tag{12}$$

Then, we have

$$\mathfrak{T}_3 = \frac{1}{2\pi i}\int_{\ell} \frac{\prod_{j=1}^{\mathcal{M}} \Gamma(b_j - \beta_j\zeta) \prod_{j=1}^{\mathcal{N}} \{\Gamma(1-a_j+\alpha_j\zeta)\}^{\mathcal{A}_j}}{\prod_{j=\mathcal{M}+1}^{\mathcal{Q}} \{\Gamma(1-b_j+\beta_j\zeta)\}^{\mathcal{B}_j} \prod_{j=\mathcal{N}+1}^{\mathcal{P}} \Gamma(a_j - \alpha_j\zeta)}$$

$$\times \frac{\Gamma(\rho+1+\sigma\zeta)}{\Gamma(1+\eta-\lambda-\lambda\rho-\lambda\sigma\zeta)} x^{\zeta}\, d\zeta$$

$$\mathfrak{T}_3 = \bar{\mathcal{H}}_{\mathcal{P}+2,\mathcal{Q}+1}^{\mathcal{M},\mathcal{N}+1}$$

$$\left[z \left| \begin{matrix} (-\rho, \sigma; 1), (1+\eta-\lambda-\lambda\rho, \lambda\sigma), (a_j, \alpha_j; \mathcal{A}_j)_{1,\mathcal{N}}, (a_j, \alpha_j)_{\mathcal{N}+1,\mathcal{P}} \\ (b_j, \beta_j)_{1,\mathcal{M}}, (b_j, \beta_j; \mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{matrix} \right. \right] \tag{13}$$

which provided $|\arg z| < \frac{1}{2}T\pi, \sigma > 0, 0 < \lambda < 1$ and $\Re(\rho+1) > 0$.

4. INTEGRAL FORMULA INVOLVING PRODUCT OF $\bar{\mathcal{H}}$-FUNCTION WITH HYPERGEOMETRIC FUNCTIONS

The hypergeometric function defined by [19]:

$$\mathbb{F}(p,q;r;z) = 1 + \sum_{k=0}^{\infty} \frac{(p)_k\,(q)_k}{(r)_k}\frac{z^k}{k\,!} \qquad (14)$$

where $(p)_k\,(q)_k$ and $(r)_k$ are of the Pochhammer symbols defined in (7).

In this section, we derive an integral formula involving product of $\bar{\mathcal{H}}$-function and hypergeometric function.

1.4. *Fourth Integral*

$$\mathfrak{T}_4 = \int_1^{\infty} x^{-\wp}\,(x-1)^{\delta-1}\; {}_2\mathbb{F}_1\left[\begin{matrix}\delta+v-\wp,\lambda+\delta-\wp\\ \delta\end{matrix};(1-x)\right] \times \bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}(z\,x)\,dx$$

$$= \frac{1}{2\pi i}\int_{\ell} \Xi\,(\zeta)\,z^{\zeta}\left\{\int_1^{\infty} x^{-\wp+\zeta}\,(x-1)^{\delta-1} \times {}_2\mathbb{F}_1\left[\begin{matrix}\delta+v-\wp,\lambda+\delta-\wp\\ \delta\end{matrix};(1-x)\right]dx\right\}d\zeta$$

$$\mathfrak{T}_4 = \frac{1}{2\pi i}\int_{\ell} \Xi\,(\zeta)\,z^{\zeta} \times\left\{\sum_{k=0}^{\infty}\frac{(-1)^k\,(\delta+v-\wp)_k\,(\lambda+\delta-\wp)_k}{(\delta)_k\,k\,!}\right\} \times\left[\int_0^{\infty} x^{-\wp+\zeta}\,(x-1)^{\delta+k-1}\,dx\right]d\zeta \qquad (15)$$

If we substitute $x = t+1 \to dx = dt$ and using the following relation

$$\int_0^{\infty} x^{a-1}\,(1+x)^{-(a+b)}\,dx = \int_0^{\infty} x^{b-1}\,(x-1)^{-(a+b)}\,dx = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \qquad (16)$$

Then integral (15) reduces to

$$\mathfrak{T}_4 = \sum_{k=0}^{\infty}\frac{(-1)^k\,(\delta+v-\wp)_k\,(\lambda+\delta-\wp)_k}{(\delta)_k\,k\,!}$$

$$\frac{1}{2\pi i}\int_{\ell}\frac{\prod_{j=1}^{\mathcal{M}}\Gamma\left(b_j-\beta_j\zeta\right)\prod_{j=1}^{\mathcal{N}}\left\{\Gamma(1-a_j+\alpha_j\zeta)\right\}^{\mathcal{A}_j}}{\prod_{j=\mathcal{M}+1}^{\mathcal{Q}}\left\{\Gamma(1-b_j+\beta_j\zeta)\right\}^{\mathcal{B}_j}\prod_{j=\mathcal{N}+1}^{\mathcal{P}}\Gamma\left(a_j-\alpha_j\zeta\right)}$$

$$\times\frac{\Gamma(\delta+k)\Gamma(\wp-\delta-k-\zeta)}{\Gamma(\wp-\zeta)}z^{\zeta}\,d\zeta$$

or

$$\mathfrak{T}_4 = \Gamma(\delta+k)$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\,(\delta+v-\wp)_k\,(\lambda+\delta-\wp)_k}{(\delta)_k\,k\,!}\times\bar{\mathcal{H}}_{\mathcal{P}+1,\mathcal{Q}+1}^{\mathcal{M}+1,\mathcal{N}}$$

$$\left[z\left|\begin{matrix}\left(a_j,\alpha_j;\mathcal{A}_j\right)_{1,\mathcal{N}},(\wp,1),\left(a_j,\alpha_j\right)_{\mathcal{N}+1,\mathcal{P}}\\ (\wp-\delta-k,1),\left(b_j,\beta_j\right)_{1,\mathcal{M}},\left(b_j,\beta_j;\,\mathcal{B}_j\right)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right.\right] \qquad (17)$$

which provided $|\arg z| < \frac{1}{2}T\pi$.

4. INTEGRAL FORMULAS INVOLVING PRODUCT OF $\bar{\mathcal{H}}$-FUNCTION AND LEGENDRE FUNCTION

The Legendre functions are the solution of Legendre's differential equation [14]

$$(1-z^2)\frac{d^2g}{dz^2} - 2z\frac{dg}{dz} + [\upsilon(\upsilon+1) - \mu^2(1-z^2)^{-1}]g = 0 \qquad (18)$$

which z,μ,υ are unrestricted.

If we set $g = (1-z^2)^{\frac{1}{2\mu}}\upsilon$, then (18) reduces to

$$(1-z^2)\frac{d^2\upsilon}{dz^2} - 2(\mu+1)z\frac{d\upsilon}{dz} + [\upsilon(\mu-\upsilon)(\mu+\upsilon+1)] = 0 \qquad (19)$$

and with $\xi = \frac{1}{2} - \frac{1}{2}z$ as the independent variable the above differential equation becomes as following

$$\xi(1-\xi)\frac{d^2v}{d\xi^2} + (\mu+1)(1-2\xi)\frac{dv}{d\xi} + [\upsilon(\mu-\upsilon)(\mu+\upsilon+1)] = 0 \qquad (20)$$

The solution of (18) in the form of Gauss hypergeometric type equation with $\alpha = (\mu - v), \beta = (\mu + v + 1)$ and $\gamma = (\mu + 1)$ as follows

$$g = \mathbb{P}_v^\mu(y) = \frac{1}{\Gamma(1-\mu)} \left(\frac{y+1}{y-1}\right)^{\frac{1}{2\mu}} \times \mathbb{F}\left[-v, v+1; 1-\mu; \frac{1}{2} - \frac{1}{2}y\right] \quad (21)$$

where $\mathbb{P}_v^\mu(y)$ is known as the Legendre function of the first kind (see [14]).

Now we establish the integrals with Legendre function.

1.5. *Fifth Integral*

$$\mathfrak{T}_5 = \int_0^1 y^{\lambda-1}\ (1-y^2)^{\frac{\mu}{2}}\, \mathbb{P}_v^\mu(y)\ \times \bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}[zy^\rho]dy$$
$$= \frac{1}{2\pi i}\int_\ell \Xi(\zeta)\, z^\zeta \times \left\{\int_0^1 y^{\lambda+\rho\zeta-1}\ (1-y^2)^{\frac{\mu}{2}}\, \mathbb{P}_v^\mu(y)dy\right\} d\zeta \quad (22)$$

Using the following integral formula [13]:

$$\int_0^1 y^{\lambda-1}\ (1-y^2)^{\frac{\mu}{2}}\, \mathbb{P}_v^\mu(y)dy = \frac{(-1)^\mu\, 2^{-\lambda-\mu}\sqrt{\pi}\,\Gamma(\lambda)\Gamma(1+\mu+v)}{\Gamma(1-\mu+v)\Gamma\left(\frac{1}{2}+\frac{\lambda}{2}+\frac{\mu}{2}-\frac{v}{2}\right)\ \Gamma\left(1+\frac{\lambda}{2}+\frac{\mu}{2}+\frac{v}{2}\right)} \quad (23)$$

where $(\mathcal{R}(\lambda) > 0, \mu \in \mathbb{N})$

Applying Eq. (23) in Eq. (22), we get

$$\mathfrak{T}_5 = (-1)^\mu\, 2^{-\lambda-\mu}\sqrt{\pi}\,\frac{\Gamma(1+\mu+v)}{\Gamma(1-\mu+v)} \times \frac{1}{2\pi i} \int_\ell \Xi(\zeta) \frac{\Gamma(\lambda+\rho\zeta)\ (2^{-\rho}z\ )^\zeta}{\Gamma\left(\frac{1}{2}+\frac{\lambda+\rho\zeta}{2}+\frac{\mu}{2}-\frac{v}{2}\right)\ \Gamma\left(1+\frac{\lambda+\rho\zeta}{2}+\frac{\mu}{2}+\frac{v}{2}\right)}\ d\zeta$$

Now we express the $\bar{\mathcal{H}}-$ function in terms of Mellin-Bernes type of contour integral by (1), we finally arrive at

$$\mathfrak{T}_5 = (-1)^\mu\, 2^{-\lambda-\mu}\sqrt{\pi}\,\frac{\Gamma(1+\mu+v)}{\Gamma(1-\mu+v)}$$
$$\times \bar{\mathcal{H}}_{\mathcal{P}+1,\mathcal{Q}+2}^{\mathcal{M},\mathcal{N}+1}\left[2^{-\rho}z\left|\begin{matrix}(1-\lambda,\rho;1),(a_j,\alpha_j;\mathcal{A}_j)_{1,\mathcal{N}}, & (a_j,\alpha_j)_{\mathcal{N}+1,\mathcal{P}} \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ (b_j,\beta_j)_{1,\mathcal{M}},\left(\frac{1}{2}-\frac{\mu}{2}+\frac{v}{2}-\frac{\lambda}{2},\frac{\rho}{2};1\right), & \left(-\frac{\lambda}{2}-\frac{\mu}{2}-\frac{v}{2},\frac{\rho}{2};1\right),(b_j,\beta_j;\ \mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right.\right] \quad (24)$$

which provide $|arz\,(z)| < \frac{1}{2}T\pi, \mathcal{R}(\lambda) > 0$ and $\mu \in \mathbb{N} - \{0\}$.

1.6. *Sixth Integral*

$$\mathfrak{T}_6 = \int_0^1 x^{\lambda-1}\ (1-x^2)^{-\frac{\mu}{2}}\, \mathbb{P}_v^\mu(x)\ \times \bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}[zx^\tau]dx$$

$$= \frac{1}{2\pi i}\int_\ell \Xi(\zeta)\, z^\zeta \times \left\{\int_0^1 y^{\lambda+\tau\zeta-1}\ (1-y^2)^{-\frac{\mu}{2}}\, \mathbb{P}_v^\mu(x)dx\right\} d\zeta \quad (25)$$

Next, using the integral formula (23), then the above integral becomes

$$\mathfrak{T}_6 = \frac{1}{2\pi i} \int_\ell \Xi(\zeta) \frac{2^{-\lambda+\mu}\sqrt{\pi}\ \Gamma(\lambda+\rho\tau)\ (2^{-\tau}z\ )^\zeta}{\Gamma\left(\frac{1}{2}+\frac{\lambda+\tau\zeta}{2}-\frac{\mu}{2}-\frac{v}{2}\right)\Gamma\left(1+\frac{\lambda+\tau\zeta}{2}-\frac{\mu}{2}-\frac{v}{2}\right)} d\zeta$$

In view of the definition (1), finally arrive at result

$$\mathfrak{T}_6 = 2^{-\lambda+\mu}\sqrt{\pi}$$
$$\times \bar{\mathcal{H}}_{\mathcal{P}+1,\mathcal{Q}+2}^{\mathcal{M},\mathcal{N}+1}\left[2^{-\tau}z\left|\begin{matrix}(1-\lambda,\tau;1),(a_j,\alpha_j;\mathcal{A}_j)_{1,\mathcal{N}}, & (a_j,\alpha_j)_{\mathcal{N}+1,\mathcal{P}} \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ (b_j,\beta_j)_{1,\mathcal{M}},\left(\frac{1}{2}+\frac{\mu}{2}+\frac{v}{2}-\frac{\lambda}{2},\frac{\tau}{2};1\right), & \left(-\frac{\lambda}{2}+\frac{\mu}{2}+\frac{v}{2},\frac{\tau}{2};1\right),(b_j,\beta_j;\ \mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right.\right] \quad (26)$$

which provided $|arz\,(z)| < \frac{1}{2}T\pi, \mathcal{R}(\lambda) > 0$ and $\mu \in \mathbb{N} - \{0\}$.

5. INTEGRAL FORMULAS INVOLVING PRODUCT OF $\bar{\mathcal{H}}$-FUNCTION WITH JACOBI POLYNOMIALS

The Jacobi polynomial [19] $\mathcal{P}_k^{(\alpha,\beta)}(x)$ is defined as

$$\mathcal{P}_k^{(\alpha,\beta)}(x) = \frac{(1+\alpha)_k}{k!}\ {}_2\mathbb{F}_1\left[\begin{matrix}-k,(1+\alpha+\beta+k) \\ (1+\alpha)\end{matrix};\frac{(1-x)}{2}\right] \quad (27)$$

where ${}_2\mathbb{F}_1$ is the classical hypergeometric functions. When $\alpha=\beta=0$, then the polynomial (27) reduces to the Legendre polynomials [see, 19].

The following integral formulas involving the product of $\bar{\mathcal{H}}$-function and Jacobi polynomials will be evaluated in this section.

1.7. ***Seventh Integral***

$$\mathfrak{T}_7=\int_{-1}^{1} y^{\lambda}\;(1-y)^{\alpha}(1+y)^{\mu}\;\mathcal{P}_k^{(\alpha,\beta)}(y)$$

$$\times\bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}[z(1+y)^l]dy$$

$$=\frac{1}{2\pi i}\int_{\ell}\Xi\,(\zeta)\,z^{\zeta}$$

$$\times\left\{\int_{-1}^{1} y^{\lambda}\;(1-y)^{\alpha}(1+y)^{\mu+\zeta l}\;\mathcal{P}_k^{(\alpha,\beta)}(y)dy\right\}d\zeta$$

Now we use the following result

$$\mathfrak{T}_7=\int_{-1}^{1} y^{\lambda}\;(1-y)^{\alpha}(1+y)^{\mu}\,\mathcal{P}_k^{(\alpha,\beta)}(y)dy$$

$$=(-1)^{\mu}\,\frac{2^{\alpha+\mu+1}\Gamma(\mu+1)\Gamma(k+\alpha+1)\Gamma(\mu+\beta+1)}{k!\;\Gamma(\mu+\alpha+k+2)\Gamma(\mu+\beta+k+1)}$$

$$\times\,{}_3\mathbb{F}_2\left[\begin{matrix}-\lambda,(\mu+\beta+1),(\mu+1)\\(\mu+\alpha+k+2),(\mu+\beta+k+1)\end{matrix};1\right]\quad(28)$$

where $\alpha>-1$ and $\beta>-1$.

Then, the above integral becomes

$$\mathfrak{T}_7=\frac{1}{2\pi i}\int_{\ell}\;\Xi\,(\zeta)\,z^{\zeta}\,(-1)^{\mu}\;2^{\alpha+\mu+l\zeta+1}$$

$$\times\frac{\Gamma(\mu+l\zeta+1)\Gamma(k+\alpha+1)\Gamma(\mu+l\zeta+\beta+1)}{k!\;\Gamma(\mu+l\zeta+\alpha+k+2)\Gamma(\mu+l\zeta+\beta+k+1)}$$

$${}_3\mathbb{F}_2\left[\begin{matrix}-\lambda,(\mu+l\zeta+\beta+1),(\mu+l\zeta+1);\\(\mu+l\zeta+\beta+k+1),(\mu+l\zeta+\alpha+k+2);\end{matrix}1\right]d\zeta$$

$$\mathfrak{T}_7=\frac{(-1)^{\mu}\,2^{\alpha+\mu+1}\Gamma(k+\alpha+1)}{k!}\sum_{r=0}^{\infty}\frac{(-\lambda)_r\;(1)^r}{r!}$$

$$\times\frac{1}{2\pi i}\int_{\ell}\frac{\prod_{j=1}^{\mathcal{M}}\Gamma(b_j-\beta_j\zeta)\;\prod_{j=1}^{\mathcal{N}}\{\Gamma(1-a_j+\alpha_j\zeta)\}^{\mathcal{A}_j}}{\prod_{j=\mathcal{M}+1}^{\mathcal{Q}}\{\Gamma(1-b_j+\beta_j\zeta)\}^{\mathcal{B}_j}\;\prod_{j=\mathcal{N}+1}^{\mathcal{P}}\Gamma(a_j-\alpha_j\zeta)}$$

$$\times\frac{\Gamma(1-(\mu-r)+l\zeta)}{\Gamma(1-(-1-\mu-\alpha-k-r)+l\zeta)}$$

$$\frac{\Gamma(1-(\mu-\beta-r)+l\zeta)}{\Gamma(1-(-\mu-\beta-k-r)+l\zeta)}\;(z2^l)^{\zeta}d\zeta$$

$$\mathfrak{T}_7=\frac{(-1)^{\mu}\,2^{\alpha+\mu+1}\Gamma(k+\alpha+1)}{k!}\sum_{r=0}^{\infty}\frac{(-\lambda)_r\;(1)^r}{r!}$$

$$\times\bar{\mathcal{H}}_{\mathcal{P}+2,\mathcal{Q}+2}^{\mathcal{M},\mathcal{N}+2}\left[z2^l\left|\begin{matrix}(\mu-\beta-r,l;1),(\mu-r,l;1),(a_j,\alpha_j;\mathcal{A}_j)_{1,\mathcal{N}},(a_j,\alpha_j)_{\mathcal{N}+1,\mathcal{P}}\\(b_j,\beta_j)_{1,\mathcal{M}},(-\mu-\beta-k-r,l;1),(-1-\mu-\alpha-k-r,l;1),(b_j,\beta_j;\mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right.\right]\quad(29)$$

which provided that $\alpha>-1,\beta>-1,\mathcal{R}(\lambda)>-1$ and $|argz|<\frac{1}{2}\pi T$.

1.8. ***Eight Integral***

$$\mathfrak{T}_8=\int_{-1}^{1}(1-x)^{l_1}(1+x)^{l_2}\,\mathcal{P}_n^{(\mu,v)}(x)$$

$$\times\bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}\left[\mathcal{Z}(1-x)^{\lambda}(1+x)^{h}\right]dx$$

$$=\frac{1}{2\pi i}\int_{\ell}\Xi\,(\zeta)\,\mathcal{Z}^{\zeta}$$

$$\times\left\{\int_{-1}^{1}\mathcal{P}_n^{(\mu,v)}(x)\;(1-x)^{l_1+\lambda\zeta}(1+x)^{l_2+h\zeta}dx\right\}d\zeta$$

Using the definition (27), we have

$$\mathfrak{T}_8=\frac{1}{2\pi i}\int_{\ell}\Xi\,(\zeta)\,\mathcal{Z}^{\zeta}\left\{\int_{-1}^{1}\;(1-x)^{l_1+\lambda\zeta}\;(1+x)^{l_2+h\zeta}\right.$$

$$\left.\frac{(1+\mu)_n}{n!}\sum_{k=0}^{\infty}\frac{(-n)_k(1+\mu+v+n)_k}{(1+\mu)_k}\frac{(1-x)^k}{2^k k!}\;dx\right\}d\zeta$$

$$\mathfrak{T}_8=$$

$$\frac{(1+\mu)_n}{n!}\sum_{k=0}^{\infty}\frac{(-n)_k(1+\mu+v+n)_k}{2^k k!\;(1+\mu)_k}\times\frac{1}{2\pi i}\int_{\ell}\Xi\,(\zeta)\,\mathcal{Z}^{\zeta}$$

$$\left\{\int_{-1}^{1}\;(1-x)^{1+(l_1+\lambda\zeta+k-1)}(1+x)^{1+(l_2+h\zeta-1)}\,dx\right\}d\zeta$$

Next, we use the following formula [19]

$$\int_{-1}^{1} (1-x)^{k+\alpha}\,(1+x)^{k+\beta}dx = 2^{2k+\alpha+\beta+1}\mathcal{B}(1+\alpha+k,1+\beta+k) \quad (30)$$

and making use of integral (1), we get

$$\mathfrak{T}_8 = \frac{2^{l_1+l_2+1}(1+\mu)_n}{n!}\sum_{k=0}^{\infty}\frac{(-n)_k(1+\mu+v+n)_k}{2^k k!\,(1+\mu)_k} \frac{1}{2\pi i}\int_{\ell}\frac{\prod_{j=1}^{\mathcal{M}}\Gamma(b_j-\beta_j\zeta)\prod_{j=1}^{\mathcal{N}}\{\Gamma(1-a_j+\alpha_j\zeta)\}^{\mathcal{A}_j}}{\prod_{j=\mathcal{M}+1}^{\mathcal{Q}}\{\Gamma(1-b_j+\beta_j\zeta)\}^{\mathcal{B}_j}\prod_{j=\mathcal{N}+1}^{\mathcal{P}}\Gamma(a_j-\alpha_j\zeta)}$$

$$\times\frac{\Gamma(1+l_1+\lambda\zeta+k)\Gamma(1+l_2+h\zeta)}{\Gamma(2+l_1+l_2+k+(\lambda+h)\zeta)}\left(2^{(\lambda+h)}\mathcal{Z}\right)^{\zeta}d\zeta$$

or

$$\mathfrak{T}_8 = \frac{(1+\mu)_n}{n!}\,2^{l_1+l_2+1}\sum_{k=0}^{\infty}\frac{(-n)_k(1+\mu+v+n)_k}{2^k k!\,(1+\mu)_k} \times\bar{\mathcal{H}}_{\mathcal{P}+2,\mathcal{Q}+1}^{\mathcal{M},\mathcal{N}+2}\left[2^{(\lambda+h)}\mathcal{Z}\left|\begin{matrix}(-l_1-k,\lambda;1),(-l_2,h;1),(a_j,\alpha_j;\mathcal{A}_j)_{1,\mathcal{N}},(a_j,\alpha_j)_{\mathcal{N}+1,\mathcal{P}}\\ (b_j,\beta_j)_{1,\mathcal{M}},(-1-l_1-l_2-k,\lambda+h;1),(b_j,\beta_j;\mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right.\right] \quad (31)$$

which provided that $\mathcal{R}(v)>-1,\mathcal{R}(\mu)>-1$ and $|argz|<\frac{1}{2}\pi T$.

1.9. *Ninth Integral*

$$\mathfrak{T}_9 = \int_{-1}^{1}(1-t)^{\lambda_1}(1+t)^{\lambda_2}\,\mathcal{P}_n^{(a,b)}(t) \times\bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}[\mathcal{Z}(1+t)^{-l}]\,dt$$

$$= \int_{-1}^{1}(1-t)^{\lambda_1}(1+t)^{\lambda_2}\,\frac{(1+a)_n}{n!} \times\sum_{k=0}^{\infty}\frac{(-n)_k(1+a+b+n)_k}{2^k k!\,(1+a)_k}(1-t)^k \times\left\{\frac{1}{2\pi i}\int_{\ell}\Xi(\zeta)\,\mathcal{Z}^{\zeta}\,(1+t)^{-l\zeta}d\zeta\right\}dt$$

Upon interchanging the order of integration and summation in above equation, we get

$$\mathfrak{T}_9 = \frac{(1+a)_n}{n!}\sum_{k=0}^{\infty}\frac{(-n)_k(1+a+b+n)_k}{2^k k!\,(1+a)_k}\int_{\ell}\frac{1}{2\pi i}\Xi(\zeta)\,\mathcal{Z}^{\zeta} \times\left\{\int_{-1}^{1}(1-t)^{1+(\lambda_1+k-1)}(1+t)^{1+(\lambda_2-l\zeta-1)}dt\right\}d\zeta$$

Using the easily-derivable result (30) and interpreting the involved Mellin-Barnes integral in term of $\overline{\mathrm{H}}$-function, we get

$$\mathfrak{T}_9 = \frac{(1+a)_n}{n!}\,2^{\lambda_1+\lambda_2+1}\sum_{k=0}^{\infty}\frac{(-n)_k(1+a+b+n)_k}{k!\,(1+a)_k} \times\Gamma(1+\lambda_1+k)\times\bar{\mathcal{H}}_{\mathcal{P}+1,\mathcal{Q}+1}^{\mathcal{M}+1,\mathcal{N}}\left[2^{-l}\mathcal{Z}\left|\begin{matrix}(a_j,\alpha_j;\mathcal{A}_j)_{1,\mathcal{N}},(1+\lambda_2,l),\,(a_j,\alpha_j)_{\mathcal{N}+1,\mathcal{P}}\\(2+\lambda_1+\lambda_2+k,l),(b_j,\beta_j)_{1,\mathcal{M}},(b_j,\beta_j;\,\mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right.\right] \quad (32)$$

The following conditions are satisfied:

(i) $\mathcal{R}(a)>-1,\mathcal{R}(b)>-1$ and $|\arg\mathcal{Z}|<\frac{1}{2}\mathrm{T}\pi$,

(ii) $\mathcal{R}\left(\lambda_1+l\,min\left(\frac{b_j}{\beta_j}\right)\right)>-1\ \ (j=1,\ldots,m)$.

1.10. *Tenth Integral*

$$\mathfrak{T}_{10} = \int_{-1}^{1}(1-t)^{\gamma_1}(1+t)^{\gamma_2}\,\mathcal{P}_n^{(a,b)}(t) \times\bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}[\mathcal{Z}(1-t)^{l}(1+t)^{-h}]\,dt$$

$$= \int_{-1}^{1}(1-t)^{\gamma_1}(1+t)^{\gamma_2}\,\frac{(1+a)_n}{n!} \times{}_2\mathbb{F}_1\left[\begin{matrix}-n,(1+a+b+n);\\(1+a);\end{matrix}\frac{(1-x)}{2}\right] \times\left\{\frac{1}{2\pi i}\int_{\ell}\Xi(\zeta)\,\mathcal{Z}^{\zeta}\,(1-t)^{l\zeta}(1+t)^{-h\zeta}dt\right\}d\zeta$$

$$= \frac{(1+a)_n}{n!}\sum_{k=0}^{\infty}\frac{(-n)_k(1+a+b+n)_k}{2^k k!\,(1+a)_k}\frac{1}{2\pi i}\int_{\ell}\Xi(\zeta)\,\mathcal{Z}^{\zeta} \left\{\int_{-1}^{1}(1-t)^{1+(\gamma_1+l\zeta+k-1)}(1+t)^{1+(\gamma_2-h\zeta-1)}dt\right\}d\zeta$$

By applying (30) and express the $\overline{\mathrm{H}}$-function of one variable in its integrand as Mellin-Barnes integral (1), we obtain

$$\mathfrak{T}_{10} = 2^{\gamma_1+\gamma_2+1}\frac{(1+a)_n}{n!}\sum_{k=0}^{\infty}\frac{(-n)_k(1+a+b+n)_k}{k!\,(1+a)_k}$$

$$\times \bar{\mathcal{H}}_{\mathcal{P}+1,\mathcal{Q}+1}^{\mathcal{M}+1,\mathcal{N}+1}\left[2^{l-h}\mathcal{Z}\left|\begin{matrix}(-\gamma_1-k,l;1),(a_j,\alpha_j;\mathcal{A}_j)_{1,\mathcal{N}}, \\ (b_j,\beta_j)_{1,\mathcal{M}},(1+\gamma_2,h;1),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j,\alpha_j)_{\mathcal{N}+1,\mathcal{P}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. \\ (-1-\gamma_1-\gamma_2-k,l-h;1),(b_j,\beta_j;\mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right] \quad (33)$$

which provided that:
(i) $\mathcal{R}(a)>-1, \mathcal{R}(b)>-1$ and $|\arg\mathcal{Z}|<\frac{1}{2}\mathrm{T}\pi$,
(ii) $\mathcal{R}\left(\gamma_1+l\,min\left(\frac{b_j}{\beta_j}\right)\right)>-1$,
and $\mathcal{R}\left(\gamma_2+l\,min\left(\frac{b_j}{\beta_j}\right)\right)>-1, \ (j=1,\dots,m)$.

1.11. *Eleventh Integral*

$$\mathfrak{T}_{11}=\int_{-1}^{1}(1-y)^{\gamma_1}(1+y)^{\gamma_2}\,\mathcal{P}_n^{(\lambda,\mu)}(y)\mathcal{P}_m^{(\rho,\sigma)}(y)$$

$$\bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}[\mathcal{Z}(1-y)^l\,]\,dy \times \frac{1}{2\pi i}\int_{\ell}\Xi(\zeta)\,\mathcal{Z}^{\zeta}$$

$$\left\{\int_{-1}^{1}(1-y)^{\gamma_1+l\zeta}(1+y)^{\gamma_2}\mathcal{P}_n^{(\lambda,\mu)}(y)\mathcal{P}_m^{(\rho,\sigma)}(y)dy\right\}d\zeta$$

Using the definition (27), we have

$$\mathfrak{T}_{11}=\frac{(1+\rho)_n}{m!}\frac{(1+\lambda)_n}{n!} \times \sum_{k=0}^{\infty}\frac{(-m)_k(-n)_k(1+\rho+\sigma+m)_k}{2^{2k}(k!)^2\,(1+\rho)_k(1+\lambda)_k}$$

$$\frac{(1+\lambda+\mu+n)_k}{\Gamma(1+\lambda+k)} \times \frac{1}{2\pi i}\int_{\ell}\Xi(\zeta)\,\mathcal{Z}^{\zeta}$$

$$=\left\{\int_{-1}^{1}(1-y)^{(1+\gamma_1+l\zeta+2k)-1}\,(1+y)^{(1+\gamma_2)-1}\,dy\right\}d\zeta$$

By applying (29) and using definition the $\overline{\mathrm{H}}$-function (1), then we arrive at

$$\mathfrak{T}_{11}=\ 2^{\gamma_1+\gamma_2+1}\frac{\Gamma(1+\rho+m)\Gamma(1+\lambda+n)}{m!\,n!}$$

$$\sum_{k=0}^{\infty}\frac{(-m)_k(-n)_k(1+\rho+\sigma+m)_k}{(k!)^2\,\Gamma(1+\rho+k)}\frac{(1+\lambda+\mu+n)_k}{\Gamma(1+\lambda+k)}$$

$$\frac{1}{2\pi i}\int_{\ell}\frac{\prod_{j=1}^{\mathcal{M}}\Gamma(b_j-\beta_j\zeta)\ \prod_{j=1}^{\mathcal{N}}\{\Gamma(1-a_j+\alpha_j\zeta)\}^{\mathcal{A}_j}}{\prod_{j=\mathcal{M}+1}^{\mathcal{Q}}\{\Gamma(1-b_j+\beta_j\zeta)\}^{\mathcal{B}_j}\prod_{j=\mathcal{N}+1}^{\mathcal{P}}\Gamma(a_j-\alpha_j\zeta)}$$

$$\times\frac{\Gamma(1+\gamma_1+2k+l\zeta)\Gamma(1+\gamma_2)}{\Gamma(2+\gamma_1+\gamma_2+l\zeta+2k)}(2^l\mathcal{Z})^{\zeta}d\zeta$$

$$=2^{\gamma_1+\gamma_2+1}\frac{\Gamma(1+\rho+m)\Gamma(1+\lambda+n)}{m!\,n!}\sum_{k=0}^{\infty}\frac{(-m)_k}{(k!)^2}$$

$$\frac{(-n)_k(1+\rho+\sigma+m)_k(1+\lambda+\mu+n)_k}{\Gamma(1+\rho+k)\Gamma(1+\lambda+k)}\Gamma(1+\gamma_2)$$

$$\times\ \bar{\mathcal{H}}_{\mathcal{P}+1,\mathcal{Q}+1}^{\mathcal{M},\mathcal{N}+1}\left[2^l\mathcal{Z}\left|\begin{matrix}(-\gamma_1-2k,l;1), \\ (b_j,\beta_j)_{1,\mathcal{M}},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j,\alpha_j;\mathcal{A}_j)_{1,\mathcal{N}}(a_j,\alpha_j)_{\mathcal{N}+1,\mathcal{P}}\dots\dots\dots \\ (-1-\gamma_1-\gamma_2-2k,l;1),(b_j,\beta_j;\mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right] \quad (34)$$

which provided that $\gamma_1>0, \mathcal{R}(\gamma_2)>-1$ and $|arg\mathcal{Z}|<\frac{1}{2}T\pi$.

6. CERTAIN INTEGRAL INVOLVING PRODUCT OF $\bar{\mathcal{H}}$-FUNCTION

For the present investigation, we need the following result of Oberhettinger [6]

$$\int_0^{\infty}x^{\varrho-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\delta}dx$$

$$=2\delta a^{-\delta}\left(\frac{a}{2}\right)^{\varrho}\frac{\Gamma(2\varrho)\Gamma(\delta-\varrho)}{\Gamma(1+\delta+\varrho)} \quad (35)$$

which provided that $0<\mathcal{R}(\varrho)<R(\delta)$.

1.12. *Twelfth Integral*

$$\mathfrak{T}_{12}=\int_0^{\infty}x^{\varrho-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\delta}$$

$$\times\ \bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}\left[\frac{y^r}{\omega^r\left(x+a+\sqrt{x^2+2ax}\right)^r}\right]dx$$

By applying (1) and the interchanging the order of integration, we get

$$\mathfrak{T}_{12}=\int_0^{\infty}x^{\varrho-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-\delta}$$

$$\times \frac{1}{2\pi i}\int_{\ell} \Xi(\zeta) \frac{y^{r\zeta}}{\omega^{r\zeta}\left(x+a+\sqrt{x^2+2ax}\right)^{r\zeta}} d\zeta dx$$

$$\mathfrak{I}_{12} = \frac{1}{2\pi i}\int_{\ell} \Xi(\zeta) \frac{y^{r\zeta}}{\omega^{r\zeta}}$$

$$\left[\int_0^\infty x^{\varrho-1}\left(x+a+\sqrt{x^2+2ax}\right)^{-(\delta+r\zeta)} dx\right] d\zeta$$

By the virtue of integral formula (35), we obtain

$$\mathfrak{I}_{12} = 2^{1-\varrho} a^{\varrho-\delta} \Gamma(2\varrho)$$
$$\frac{1}{2\pi i}\int_{\ell} \Xi(\zeta)\left(\frac{y^r}{a^r\omega^r}\right)^{\zeta} \frac{\Gamma(\delta+1+r\zeta)\Gamma(\delta-\varrho+r\zeta)}{\Gamma(\delta+r\zeta)\Gamma(1+\delta+\varrho+r\zeta)} d\zeta$$

Interpreting the involved Mellin-Barnes integral in term of $\bar{\mathcal{H}}$ -function (1), then we arrive at

$$\mathfrak{I}_{12} = 2^{1-\varrho} a^{\varrho-\delta} \Gamma(2\varrho)$$

$$\times \bar{\mathcal{H}}^{\mathcal{M},\mathcal{N}+2}_{\mathcal{P}+2,\mathcal{Q}+2}\left[\left(\frac{y^r}{a^r\omega^r}\right)\middle| \begin{matrix}(-\delta,r;1),(\varrho-\delta-1,r;1),(a_j,\alpha_j;\mathcal{A}_j)_{1,\mathcal{N}}(a_j,\alpha_j)_{\mathcal{N}+1,\mathcal{P}}\dots\dots\dots \\ (b_j,\beta_j)_{1,\mathcal{M}},(-1-\delta,\delta;1),(-\varrho-\delta,r;1),(b_j,\beta_j;\mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right] \quad (36)$$

The converge conditions for the validity of (36) is as follows:

$(i)\mathcal{R}(\varrho) > 0, \mathcal{R}(\varrho) + r\, min\, \mathcal{R}\left(\frac{b_j}{\beta_j}\right) > \max[0,(\delta+r\zeta)]$,

$(ii)\ \alpha > 0, |\arg \mathcal{Z}| < \frac{1}{2}T\pi.$

7. INTEGRAL FORMULAS INVOLVING PRODUCT OF $\bar{\mathcal{H}}$-FUNCTION WITH ALGEBRAIC FUNCTION

In this section, the generalized integral formulas involving $\bar{\mathcal{H}}$-function (1), are established here by inserting with the suitable arguments of algebraic functions.

1.13. *Thirteen Integral*

$$\mathfrak{I}_{13} = \int_1^\infty t^{-\lambda}(t-1)^{\mu-1} \bar{\mathcal{H}}^{\mathcal{M},\mathcal{N}}_{\mathcal{P},\mathcal{Q}}[zt]\, dt$$
$$= \frac{1}{2\pi i}\int_{\ell} \Xi(\zeta)\, z^{\zeta}\left[\int_0^\infty t^{-\lambda+\zeta}(t-1)^{\mu-1}\, dt\right] d\zeta$$

Putting $t = p+1$ so that $dt = dp$ and using the following known result,

$$\Gamma(x)\Gamma(y) = \Gamma(x+y)\int_0^\infty t^{x-1}(1+t)^{-(x+y)}dt$$
$$= \Gamma(x+y)\int_0^\infty t^{y-1}(1+t)^{-(x+y)}dt \quad (37)$$

then we arrive at

$$\mathfrak{I}_{13} = \frac{1}{2\pi i}\int_{\ell} \frac{\prod_{j=1}^{\mathcal{M}}\Gamma(b_j-\beta_j\zeta)}{\prod_{j=\mathcal{M}+1}^{\mathcal{Q}}\{\Gamma(1-b_j+\beta_j\zeta)\}^{\mathcal{B}_j}}$$

$$\frac{\prod_{j=1}^{\mathcal{N}}\{\Gamma(1-a_j+\alpha_j\zeta)\}^{\mathcal{A}_j}\Gamma(\mu)\Gamma(\lambda-\mu-\zeta)}{\prod_{j=\mathcal{N}+1}^{\mathcal{P}}\Gamma(a_j-\alpha_j\zeta)\ \Gamma(\lambda-\mu)} z^{\zeta}\, d\zeta$$

Now using (1), after a little simplification, we obtain the following result

$$\mathfrak{I}_{13} = \Gamma(\mu)\, \bar{\mathcal{H}}^{\mathcal{M}+1,\mathcal{N}}_{\mathcal{P}+1,\mathcal{Q}+1}$$

$$\left[z\middle| \begin{matrix}(a_j,\alpha_j;\mathcal{A}_j)_{1,\mathcal{N}},\ (a_j,\alpha_j)_{\mathcal{N}+1,\mathcal{P}},(\lambda,1) \\ (\lambda-\mu,1)(b_j,\beta_j)_{1,\mathcal{M}},(b_j,\beta_j;\ \mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right] \quad (38)$$

1.14. *Fourteenth Integral*

$$\mathfrak{I}_{14} = \int_{-1}^{1}(1-t)^{\lambda}(1+t)^{\mu}\, \bar{\mathcal{H}}^{\mathcal{M},\mathcal{N}}_{\mathcal{P},\mathcal{Q}}[z(1-t)^k]\, dt$$
$$= \frac{1}{2\pi i}\int_{\ell} \Xi(\zeta)\ z^{\zeta}$$
$$\times\left[\int_{-1}^{1}(1-t)^{(1+\lambda+k\zeta)-1}(1+t)^{(1+\mu)-1}\ dt\right] d\zeta$$

Applying the known result (30) and using (1), after straight forward calculation, we obtain the following result

$$\mathfrak{I}_{14} = 2^{\mu+\lambda+1}\,\Gamma(1+\mu)$$

$$\times \frac{1}{2\pi i}\int_{\ell} \frac{\prod_{j=1}^{\mathcal{M}}\Gamma(b_j-\beta_j\zeta)}{\prod_{j=\mathcal{M}+1}^{\mathcal{Q}}\{\Gamma(1-b_j+\beta_j\zeta)\}^{\mathcal{B}_j}}$$

$$\frac{\prod_{j=1}^{\mathcal{N}}\{\Gamma(1-a_j+\alpha_j\zeta)\}^{\mathcal{A}_j}\Gamma(1+\lambda+k\zeta)}{\prod_{j=\mathcal{N}+1}^{\mathcal{P}}\Gamma(a_j-\alpha_j\zeta)\ \Gamma(2+\mu+\lambda+k\zeta)}(2^k z)^{\zeta} d\zeta$$

$$\mathfrak{I}_{14} = 2^{\mu+\lambda+1}\,\Gamma(1+\mu)\, \bar{\mathcal{H}}^{\mathcal{M},\mathcal{N}+1}_{\mathcal{P}+1,\mathcal{Q}+1}$$

$$\left[2^k z\middle| \begin{matrix}(-\lambda,k;1),(a_j,\alpha_j;\mathcal{A}_j)_{1,\mathcal{N}},\ (a_j,\alpha_j)_{\mathcal{N}+1,\mathcal{P}} \\ (b_j,\beta_j)_{1,\mathcal{M}},(-1-\mu-\lambda,k;1),(b_j,\beta_j;\ \mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right] \quad (39)$$

1.15. *Fifteenth Integral*

$$\mathfrak{I}_{15} = \int_0^\infty y^{\eta-1}(y+k)^{-\lambda}\, \bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}[zy]\, dy$$

$$= \frac{1}{2\pi i}\int_\ell \Xi(\zeta)\, z^\zeta \left[\int_0^\infty y^{\zeta+\eta-1}(y+k)^{-\lambda}\, dy\right] d\zeta$$

Substituting $y = tk \Rightarrow dy = kdt$, then we arrive at

$$\mathfrak{I}_{15} = k^{-\eta+\lambda}\frac{1}{2\pi i}$$

$$\int_\ell \frac{\prod_{j=1}^{\mathcal{M}}\Gamma(b_j - \beta_j\zeta)\prod_{j=1}^{\mathcal{N}}\{\Gamma(1-a_j+\alpha_j\zeta)\}^{\mathcal{A}_j}}{\prod_{j=\mathcal{M}+1}^{\mathcal{Q}}\{\Gamma(1-b_j+\beta_j\zeta)\}^{\mathcal{B}_j}\prod_{j=\mathcal{N}+1}^{\mathcal{P}}\Gamma(a_j-\alpha_j\zeta)} \times \frac{\Gamma(\eta+\zeta)\Gamma(\lambda-\eta-\zeta)}{\Gamma(\lambda)}(kz)^\zeta d\zeta$$

or

$$\mathfrak{I}_{15} = \frac{k^{-\eta+\lambda}}{\Gamma(\lambda)}\bar{\mathcal{H}}_{\mathcal{P}+1,\mathcal{Q}+1}^{\mathcal{M}+1,\mathcal{N}+1}\left[kz \left| \begin{matrix}(1-\eta,1;1),(a_j,\alpha_j;\mathcal{A}_j)_{1,\mathcal{N}},(a_j,\alpha_j)_{\mathcal{N}+1,\mathcal{P}}\\ (b_j,\beta_j)_{1,\mathcal{M}},(\lambda-\eta,1),(b_j,\beta_j;\mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right.\right] \quad (40)$$

8. INTEGRAL FORMULAS INVOLVING PRODUCT OF $\bar{\mathcal{H}}$-FUNCTION WITH GENERAL CLASS OF POLYNOMIALS

The general class of polynomials $\mathfrak{S}_{n_1,\dots,n_r}^{m_1,\dots,m_r}[u]$ introduced by Srivastava [15]

$$\mathfrak{S}_{n_1,\dots,n_r}^{m_1,\dots,m_r}[u] = \sum_{\mathcal{J}_1=0}^{[n_1/m_1]}\cdots\sum_{\mathcal{J}_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_i\mathcal{J}_i}}{\mathcal{J}_i!}\mathcal{A}_{m_i,\mathcal{J}_i}\, u^{\mathcal{J}_i} \quad (41)$$

where $n_1,\dots,n_r = 0,1,2\dots; m_1,\dots,m_r$ is an arbitrary positive integers, the coefficients $\mathcal{A}_{m_i,\mathcal{J}_i}$ $(m_i,\mathcal{J}_i \geq 0)$ are arbitrary constants, real or complex. On suitably specializing the coefficients $\mathcal{A}_{m_i,\mathcal{J}_i}, \mathfrak{S}_{n_1,\dots,n_r}^{m_1,\dots,m_r}[u]$ yields a number of known polynomials as its special cases. There includes, among other, the Bessel polynomials, the Jecobi polynomials, Harmite polynomials, the Laguerre polynomials, the gould-Hopper polynomials and several others [16].

Now we derive the following integral.

1.16. *Sixteenth Integral*

$$\mathfrak{I}_{16} = \int_{-1}^{1}(1-t)^{\rho-1}(1+t)^{\sigma-1}\,\mathfrak{S}_{n_1,\dots,n_r}^{m_1,\dots,m_r}[y(1-t)^\mu(1+t)^v] \times \bar{\mathcal{H}}_{\mathcal{P},\mathcal{Q}}^{\mathcal{M},\mathcal{N}}[z(1-t)^h(1-t)^k]dt$$

$$= \int_{-1}^{1}(1-t)^{\rho-1}(1+t)^{\sigma-1}\sum_{\mathcal{J}_1=0}^{[n_1/m_1]}\cdots\sum_{\mathcal{J}_r=0}^{[n_r/m_r]} \times \prod_{i=1}^{r}\frac{(-n_i)_{m_i\mathcal{J}_i}}{\mathcal{J}_i!}\mathcal{A}_{m_i,\mathcal{J}_i}\{y(1-t)^\mu(1+t)^v\}^{\mathcal{J}_i} \times \left[\frac{1}{2\pi i}\int_\ell \Xi(\zeta)\,[z(1-t)^h(1-t)^k]^\zeta d\zeta\right]dt$$

or

$$\mathfrak{I}_{16} = \sum_{\mathcal{J}_1=0}^{[n_1/m_1]}\cdots\sum_{\mathcal{J}_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_i\mathcal{J}_i}}{\mathcal{J}_i!}\mathcal{A}_{m_i,\mathcal{J}_i} \times\frac{1}{2\pi i}\int_\ell \Xi(\zeta)z^\zeta \times\left[\int_{-1}^{1}(1-t)^{\rho+\mu\mathcal{J}_i+h\zeta-1}(1+t)^{\sigma+v\mathcal{J}_i+k\zeta-1}\,dt\right]d\zeta$$

Next we use the formula (30) and in view of the definition (1), we arrive at

$$\mathfrak{I}_{16} = 2^{\rho+\sigma-1}\sum_{\mathcal{J}_1=0}^{[n_1/m_1]}\cdots\sum_{\mathcal{J}_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_i\mathcal{J}_i}}{\mathcal{J}_i!}\mathcal{A}_{m_i,\mathcal{J}_i}\,2^{(\mu+v)\mathcal{J}_i}\,y^{\mathcal{J}_i}$$

$$\times\frac{1}{2\pi i}\int_\ell \frac{\prod_{j=1}^{\mathcal{M}}\Gamma(b_j-\beta_j\zeta)\prod_{j=1}^{\mathcal{N}}\{\Gamma(1-a_j+\alpha_j\zeta)\}^{\mathcal{A}_j}}{\prod_{j=\mathcal{M}+1}^{\mathcal{Q}}\{\Gamma(1-b_j+\beta_j\zeta)\}^{\mathcal{B}_j}\prod_{j=\mathcal{N}+1}^{\mathcal{P}}\Gamma(a_j-\alpha_j\zeta)} \times\frac{\Gamma(\rho+\mu\mathcal{J}_i+h\zeta)\Gamma(\sigma+v\mathcal{J}_i+k\zeta)}{\Gamma(\rho+\sigma+(\mu+v)\mathcal{J}_i+(h+k)\zeta)}\left(2^{(h+k)}z\right)^\zeta d\zeta$$

or

$$\mathfrak{I}_{16} = 2^{\rho+\sigma-1}\sum_{\mathcal{J}_1=0}^{[n_1/m_1]}\cdots\sum_{\mathcal{J}_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_i\mathcal{J}_i}}{\mathcal{J}_i!}\mathcal{A}_{m_i,\mathcal{J}_i}\,2^{(\mu+v)\mathcal{J}_i}\,y^{\mathcal{J}_i}$$

$$\times\bar{\mathcal{H}}_{\mathcal{P}+2,\mathcal{Q}+1}^{\mathcal{M},\mathcal{N}+2}\left[2^{(h+k)}z\left|\begin{matrix}(1-\rho-\mu\mathcal{J}_i,h;1),(1-\sigma-v\mathcal{J}_i,k;1),(a_j,\alpha_j;\mathcal{A}_j)_{1,\mathcal{N}}(a_j,\alpha_j)_{\mathcal{N}+1,\mathcal{P}}\\ (b_j,\beta_j)_{1,\mathcal{M}},(1-\rho-\sigma-(\mu+v)\mathcal{J},h+k;1_i),(b_j,\beta_j;\mathcal{B}_j)_{\mathcal{M}+1,\mathcal{Q}}\end{matrix}\right.\right] \quad (42)$$

The conditions for validity of (42) are

(i) $|\arg z| < \frac{1}{2}T\pi$,

(ii)$\rho \geq 1, \sigma \geq 1, \mu \geq 0, v \geq 0, h \geq 0, k \geq 0$ (where h and k are not both zero simultaneously),

(iii) $\mathcal{R}(\rho) + h \min\left[\mathcal{R}\left(\frac{b_j}{\beta_j}\right)\right] > 0,$

$\mathcal{R}(\sigma) + k \min\left[\mathcal{R}\left(\frac{b_j}{\beta_j}\right)\right] > 0.$

9. CONCLUDING REMARK

Recently, the investigations for extension of some special function have become important. Thus, many extensions of special functions have been obtained by authors in different studies. In this work, the authors investigated and derived integral formulas of H (bar)-function associated with hypergeometric functions, algebraic function, Srivastava polynomials, Jacobi polynomial, Legendre polynomials, Bessel function and certain class integrals. In view of closed relationship between them, it does not seem difficult to construct various integral formulas. Most of results obtained here, are significant and general in character. The provided results obtained are new and have uniqueness identity in literature. So we can find applications in the field special functions.

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